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# The Response of Six Building Shapes to Turbulent Wind

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# The response of six building shapes to turbulent wind

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The overall response of a building to wind is governed by steady and unsteady aerodynamic characteristics in winds from different directions, and its mechanical properties such as mass, stiffness, and damping. The general characteristics of the response of bluff building shapes are presented. Typical responses for six tall buildings, as well as their dependence on density and damping, are discussed. Statistical predictions of extreme responses required in design decision are determined through the integration of meteorological factors with aerodynamic and structural properties.

It is shown how building shape and structural properties can affect the overall predicted performance.

## Introduction

Although tall buildings have been traditionally designed for static loads, the turbulent effect of the winds have been recognized for many years. During the boom in skyscraper construction, in 1930 Spurr (1930), in his influential treatise on *Wind bracing*, stated somewhat pessimistically that 'the whole question of vibration in buildings from the effects of variable wind pressures is complicated by the indeterminate nature of the pressures themselves as well as by the great variation in size, shape, weight, height, and location of buildings'.

Since then, the problem of predicting vibration of buildings has become somewhat more tractible, largely through the use of statistical methods. In this paper, the dynamic and static response characteristics of a number of building shapes are examined with particular emphasis placed on the prediction of extreme responses during the life of the building.

## Theoretical and experimental approaches to turbulent wind action

Studies into the structure of wind (Davenport 1968) have suggested that the wind can be treated as a 'locally stationary random process'. This implies that the wind can be described by a stationary process $\phi(t)$ whose energy is contained in a frequency range lying above some lower limiting frequency $f_1$ and whose amplitude is modulated by another process $V(t)$ whose fluctuation rate is much slower than $\phi(t)$, that is, its energy is contained in a frequency range below $f_1$.† In this sense $\phi(t)$ describes the gust fluctuations and $V(t)$ the fluctuations in the mean wind speed. The amplitude of gust velocities, it is found, tends to be more or less proportional to the mean wind velocity. It appears that $f_1$ should be taken as approximately 1 cycle in 20 min: thus, the moving mean wind speed should be averaged over about 20 min.

Roughness of the terrain is a dominant influence on both the intensity of the turbulence as well as the variation of the mean wind speed with height; in rougher areas, the turbulent intensity is stronger and the increase of mean velocity with height more rapid.

The locally stationary character of the wind suggested that the statistical concepts of the

† Notation used in this paper appears on p. 393.

stationary random process could be applied to the problem of gust loading (Barstein 1968; Davenport 1961). In applying this approach, there were obvious difficulties associated with unsteady flow on bluff bodies as well as the spatial variations in the flow introduced by turbulence. However, some progress could be made in certain areas. If the important effects were confined to disturbances whose wavelengths were very large compared to the structure and its associated wake regions, a quasi-steady approach could be applied; at the other end of the scale, when the wavelengths were extremely small compared to the size of the structure, the influence of turbulence should be rather small.

Since the primary structural concern is with problems of resonance, it appears that the quasi-steady assumption should be applicable to structures provided that $f_0 D/\bar{V}_c \ll 1$ (where $f_0$ is the natural frequency, $D$ the diameter, and $\bar{V}$ the mean wind speed). Structures in this category included water towers and lighting towers having a relatively concentrated silhouette area and low natural frequencies.

A second category of structures could also be treated using the quasi-steady approach; these were the so-called 'line-like' structures, such as slender towers and long-span bridges, in which one dimension of the structure was slender in comparison to the other. For such structures, the flow around any cross-section could be assumed to be locally 'quasi-steady' (again providing that $f_0 D/\bar{V}_c \ll 1$) and the spatial correlations in the long direction could be inferred using the 'strip assumption' and the spatial correlations of the oncoming flow (Davenport 1962).

For such structures, loaded in the drag direction, the statistical theory has been shown to provide reasonable solutions (Vickery & Davenport 1968).

To describe the relation between the fluctuations in the oncoming flow and the resulting fluctuations in pressure, the term 'aerodynamic admittance' was introduced, denoted by $|\chi(f)|^2$. This can be defined as the ratio,

$$|\chi(f)|^2 = \frac{f S_F(f)}{\bar{F}^2} \bigg/ \frac{4 f S_U(f)}{\bar{U}^2}, \tag{1}$$

where $S_F(f)$ and $S_U(f)$ are the power spectra of force ($F$) and velocity ($U$) at frequency $f$, and $\bar{F}$ and $\bar{U}$ denote the mean force and velocity. (The factor 4 in this expression arises from the linearization of the squaıe law dependence of force on velocity and assumes that the velocity fluctuation is relatively small compared to the mean velocity.) The aerodynamic admittance defines the relative amplitude of the aerodynamic force fluctuation to the velocity fluctuation at various frequencies. For the quasi-steady situation, the aerodynamic admittance is unity; and this fact was demonstrated by Davenport (1963) from field experiments to hold for a large board in the natural wind.

For three-dimensional, spatially extended structures, it is to be expected that the aerodynamic admittance for drag fluctuations falls off with increasing reduced frequency owing to the increased 'blurring' of the fluctuations and reductions in the coherence of the forces over large areas. This was investigated by Davenport (1963) and more thoroughly by Vickery (1966) on flat plates and elementary building forms. Vickery showed that, in fact, a simple model in which the local fluctuations of aerodynamic force were estimated directly from the correlation of the oncoming velocities (the so-called 'lattice' approach) could be quite effective in predicting the response. This was later used in producing simple design approaches for tall buildings (Davenport 1966).

The question of lateral forces induced by turbulence is considerably more complex than that of the drag fluctuations; the reason for this is bound up with the dominance of the vortex shedding forces. In this, the predominant role of turbulence appears to be in dispersing the energy of the vortex shedding through a wider frequency band and to destroy its lengthwise correlation. Recently, this subject has been explored experimentally by Surry (1969); he was able to demonstrate the important influences of the scale and intensity of the turbulence on the forces generated on a circular cylinder.

Experimental and theoretical work by Bearman (1969) suggests that a fundamental understanding of the response of structures to turbulence can be extended to reduced frequency ranges for which the quasi-steady theory no longer may be expected to apply.

While the understanding of the forces themselves is of interest in building up a rational theory, it is, in fact, the response of the structure to the forces which, from the engineering viewpoint, is of primary concern. To understand this phenomenon, it is also necessary to introduce the dynamic character of the structure involved. If the statistical properties of the forces are known, it is, of course, not a particularly difficult matter in principle to predict the responses, provided that only mechanical terms are significant in the description of the dynamic system and the aerodynamic forces are purely those externally induced by the flow conditions.

This situation is found to hold under most circumstances for buildings: however, for exceptionally light structures (or alternatively very high wind speeds) aerodynamic terms can play a dominant role in the behaviour. Aerodynamic damping and stiffness terms, for example, can play an important part in the instability of suspension bridges and other structures. In turbulent flow, the presence of negative aerodynamic damping also enhances the response to turbulence (by reducing the total available damping) and can lead to large amplitudes at velocities below the onset of instability. This problem has been studied by Novak & Davenport (1970), who also showed that at reduced velocities beyond the onset of instability the amplitudes of vibration predicted by Parkinson's theory could be significantly suppressed by the turbulence.

A further effect of turbulence is to suppress the dynamic response to vortex shedding excitation; this has been noted by Scruton (1966), Davenport (1966), Rosati (1968), and others.

## Aeroelastic response of six building shapes

While the development of understanding of the wind-loading process, from velocity description to force description to response description, is of obvious fundamental importance, it can be somewhat tangential to the questions of engineering significance. For structural engineering purposes the primary focus is on the overall response prediction, since it is upon this that the design decisions must rest. This prediction must take into account not only the response to *a* wind but to *all* likely winds. To assess this question, a seemingly different approach may be necessary since the approach must be such as to integrate together the meteorological factors, the aerodynamic factors, as well as the structural response charactericts. Since winds of different velocity and direction each occur with different probabilities, it is necessary to have a knowledge of the complete aerodynamic response for all such speeds and directions.

For this purpose, the complete mapping of the dynamic and static responses can be achieved readily experimentally using the boundary layer wind tunnel (designed to simulate the turbulent flow conditions near the ground at the site of the structure). In a recent study (Davenport & Isyumov 1967, 1968), the six tall building shapes, illustrated in figure 1, were investigated. These models were of similar height, equal cross-sectional area, average building density, damping, and stiffness (see appendix). Under tests, the models were pivoted at the base by a spring system producing linear modes of vibration in the two transverse directions. The adoption of the 'linear

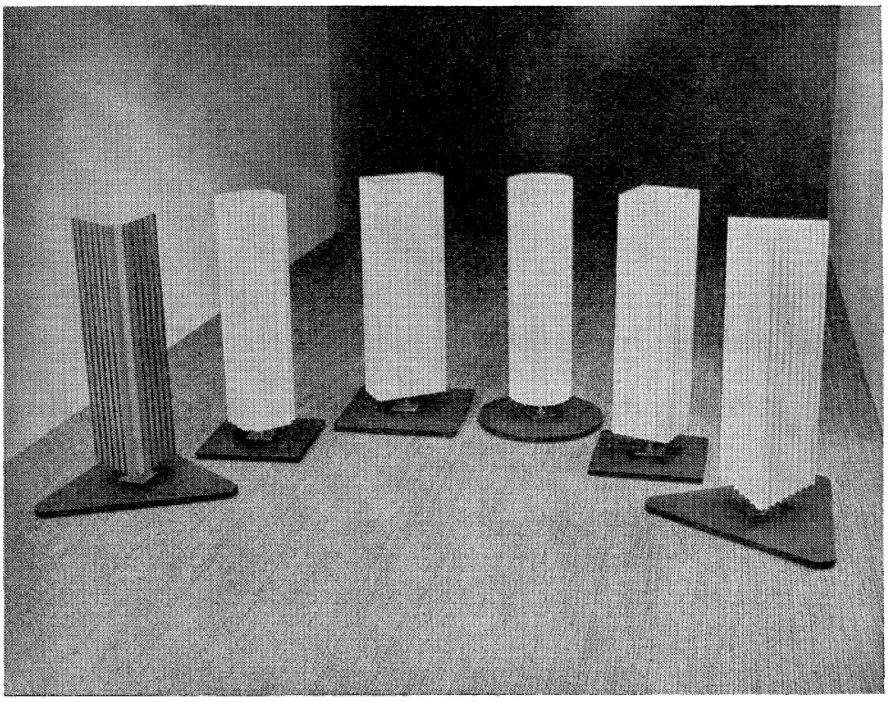

FIGURE 1. View of six building shapes studied in the boundary-layer wind-tunnel laboratory.

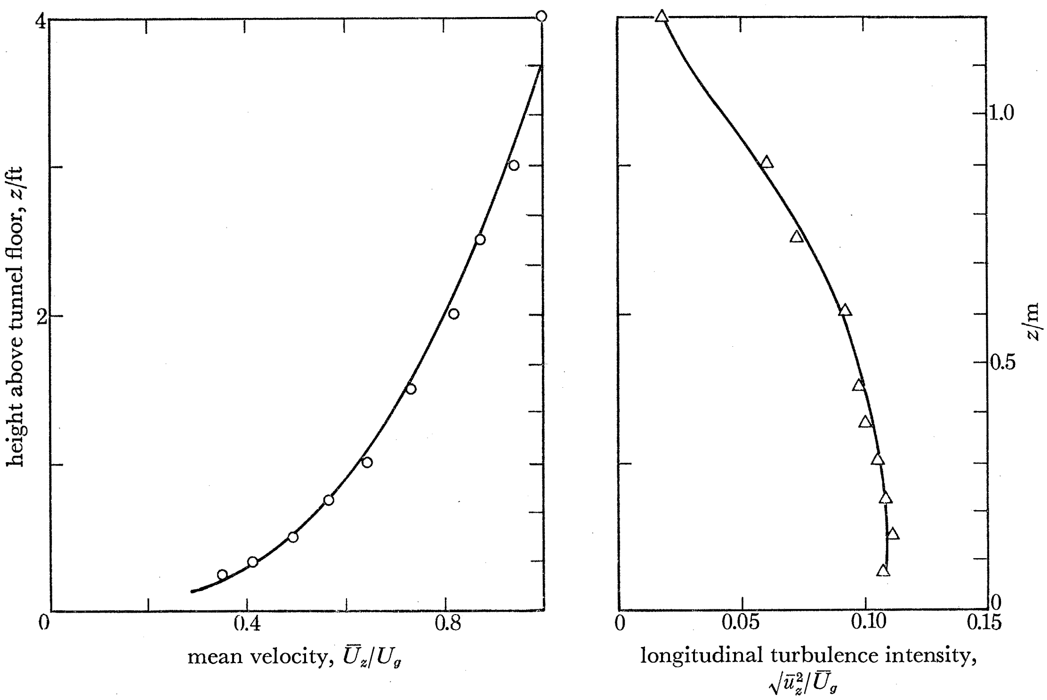


FIGURE 2. Mean velocity and turbulence intensity variations with height in boundary-layer flow. ——, theory; ○ and △ actual measurements. The theoretical line for mean velocity of wind over a built-up area was calculated from $\bar{U}_z/\bar{U}_g = (Z/Z_g)^{0.36}$.



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mode' is dictated by the essentially linear mode shape found in the fundamental transverse modes of tall buildings.

The flow properties during the experiments described are summarized in figures 2 and 3. Essentially the flow is a reasonable representation of the turbulence in a moderate size city at a scale of 1/400; at the same scale, the buildings are representative of roughly 70 storey structures.

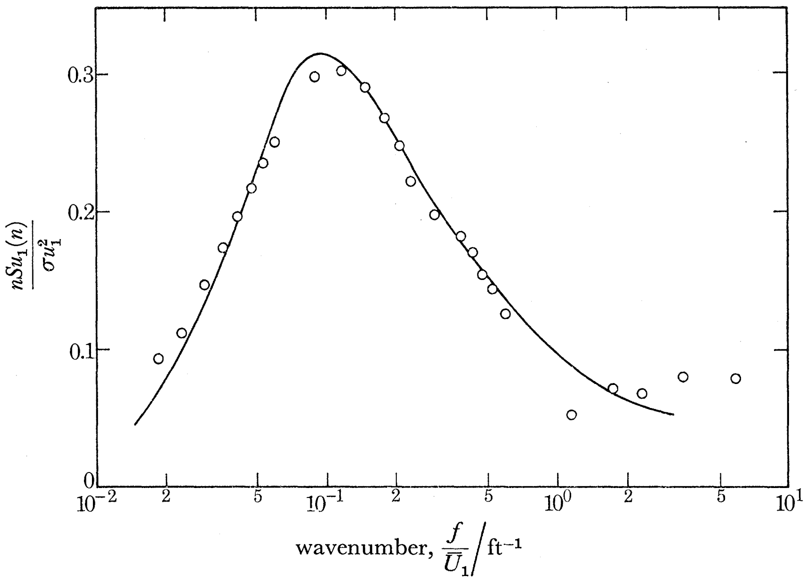


FIGURE 3. Power spectral density of longitudinal wind spectrum in boundary-layer flow. ——, Davenport's longitudinal strong wind spectrum; ○, wind tunnel measurements. Local scale: $\bar{U}_1/2\pi n_{\text{peak}} \approx 0.43$ m (1.4 ft). $z = 0.3$ m (1.0 ft).

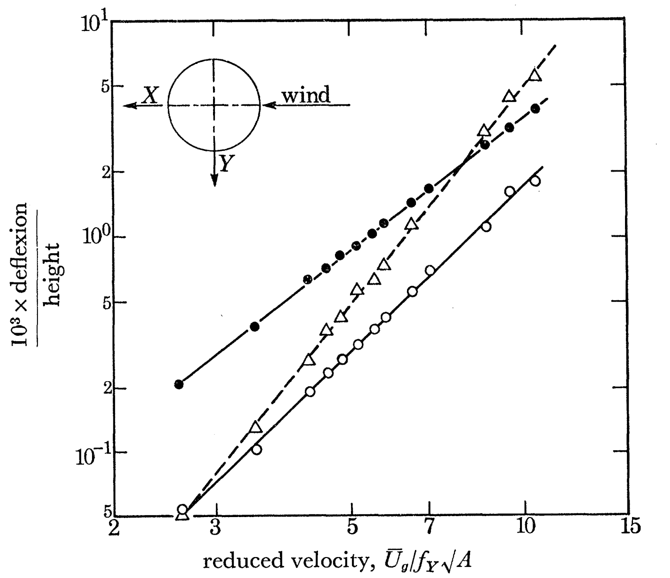


FIGURE 4. The steady and fluctuating response of the circular building shape in boundary-layer flow as a function of reduced velocity. ○—○, $\sigma_x/H$; △– – –△, $\sigma_y/H$; ●—●, $\bar{X}/H$.

The responses of the structure were characterized by measurement of the mean and the root mean square (r.m.s.) deflexions at the top expressed as a fraction of the total height. Because of the low damping, the responses were 'narrow band', that is to say confined to the resonant peak of the structure and were of Gaussian character.

A typical variation of response amplitude with velocity is shown in figure 4 for the circular cylindrical shape. This indicates that both root mean square and mean responses may be adequately represented by power law relations of the form,

$$\bar{X}/H = B_X V^2, \tag{3}$$

$$\sigma_X/H = C_X V^K, \tag{4}$$

with similar expression for $\bar{Y}$ and $\sigma_Y$.

Here,

$$V = \bar{U}_g/f_Y\sqrt{A}, \tag{5}$$

in which $\bar{U}_g$ is the mean speed at the top of the building, $f_Y$ the natural frequency in the $Y$ direction, and $A$ the cross-sectional area.

Expressed in this form it can be shown that,

$$B = \frac{1}{8\pi^2}\frac{\rho_{\text{air}}}{\rho_{\text{bldg}}}\frac{H}{\sqrt{A}}C_M, \tag{6}$$

where $C_M$ is an 'aerodynamic base moment coefficient', $\rho_{\text{air}}$ and $\rho_{\text{bldg}}$ are air and effective building density, and $H$ is the height of the building.

While equation (3) may be regarded as exact, equation (4) is an approximate form, but which is in close agreement with experiment. The value of the exponent $K$ is normally a little larger than 2, the value to be expected if the dynamic response is proportional to the mean response. The explanation for this is that at higher wind speeds the wavelength of the gusts resonating with the structure becomes larger in comparison with the structure; as a consequence, the turbulent fluctuations become better correlated and, therefore, more effective. Furthermore, the longer wavelength gusts usually contain greater energy.

The value of the dynamic amplitude $C$ is a function of the mechanical damping of the system as well as the mass. It has been shown (Davenport 1966) that provided the structural response is mainly resonant (low structural damping) and aerodynamic damping is not significant,

$$C_{X,Y} \propto \frac{1}{\sqrt{\zeta}}\frac{\rho_{\text{air}}}{\rho_{\text{bldg}}}. \tag{7}$$

This expression permits adjustment of the response for different damping and building densities. $\zeta$ is the structural damping.

A summary of the coefficients $B_X$, $B_Y$, $C_X$, $C_Y$, $K_X$ and $K_Y$ is given in figure 5. It is noted that values of $K$ lie between 2 and 4. The values of $C$ indicate sharp peaks at certain angles; it appears that these usually reflect directions for which the aerodynamic damping is negative. The mean coefficients are similar to those of Ackeret & Egli (1966).

## Integration with meteorological data

Predictions of the probabilities of occurrence of large amplitudes can be made using statistical procedures described by Davenport (1969). Let the probability density of a mean wind at azimuth $\alpha$ and wind speed $\bar{V}$ be $p(\bar{V}, \alpha)$. Suppose at azimuth $\alpha$ the velocity required for the peak $X$ displacement to equal $\hat{X}$ is $V_1(\hat{X}, \alpha)$. Then the probability of the response $\hat{X}$ being exceeded $p(>\hat{X})$ is

$$p(>\hat{X}) = \int_0^{2\pi}\int_{V_1}^{\infty} p(\bar{V}, \alpha)\,\mathrm{d}\bar{V}\,\mathrm{d}\alpha. \tag{8}$$

Now the peak value can be written $\hat{X} = \bar{X} + g\sigma_X$, (9)

where $g$ is a statistical quantity relating the largest value to the standard deviation. Since the peak distribution is usually quite narrow it is appropriate to take $g$ as the expected value, that is,

$$E(g) = \bar{g} \simeq \surd(2\ln f_0 T) + 0.5772/\surd(2\ln f_0 T), \tag{10}$$

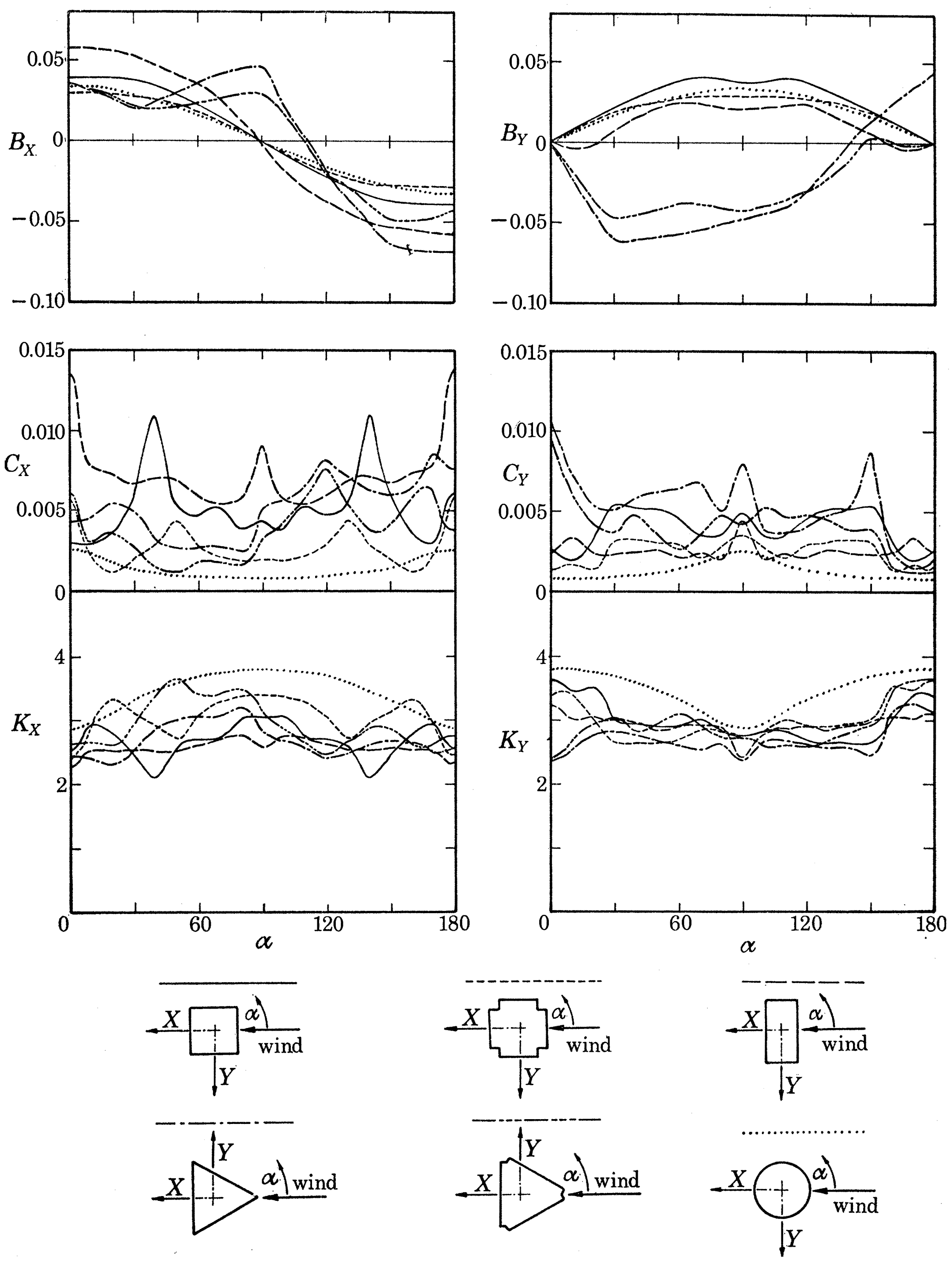


FIGURE 5. Coefficients describing the steady and fluctuating responses of six building shapes for different wind directions in turbulent boundary-layer flow.



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where $f_0$ is the natural frequency of the building, and $T$ is the sample duration (taken as the full-scale equivalent of 20 min). It is found that $g \simeq 3.5$.

Substituting from equations (3) and (4) into (9) (with the constants given in figure 5) the value $V_1 = (\hat{X}, \alpha)$ can be numerically established. The probability $p(>\hat{X})$ is then found from equation (8). For convenience, $p(>\hat{X})$ can usually be fitted by a Weibull distribution,

$$p(>\hat{X}) = \exp\{-(\hat{X}/d)^{\beta}\}. \tag{11}$$

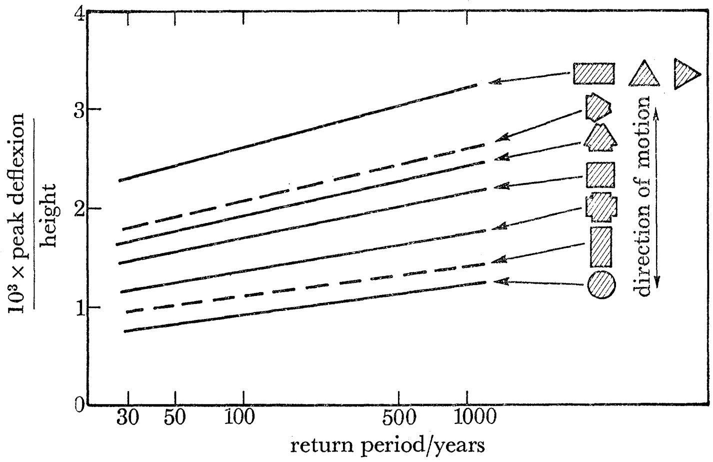


FIGURE 6. Effect of shape of cross-section on maximum deflexions of six building shapes.

The largest of these peak responses during a year is then given approximately by the type 1 extreme value distribution,

$$p(>\hat{X})_{\max} = \exp[-\exp\{-\alpha(\hat{X}_{\max}-\mu)\}], \tag{12}$$

where

$$\mu = d\{\ln n\}^{1/\beta}, \tag{13}$$

$$\frac{1}{a} = \frac{d}{\beta}\{\ln n\}^{1/\beta-1}, \tag{14}$$

and $n$ = number of effectively uncorrelated samples in the year.

It turns out (Davenport 1968) that an effective value for $n$ is about 800—the precise value is not important.

For purposes of comparison, a Rayleigh distribution of wind speed was used, defined by,

$$p(V,\alpha) = \frac{V}{2\pi V_0}\exp\left\{-\frac{V^2}{2V_0^2}\right\}, \tag{15}$$

where $V_0 = 1.07$. (Both $V$ and $V_0$ are reduced velocities.)

The maximum responses for the six building shapes determined in this way are shown in figure 6. Since all properties but the shape of the six buildings are identical, it may be concluded that significant aerodynamic penalties are associated with various shapes and performance can be improved by proper choice of shape or detailing. The influence of building density, stiffness, and damping can be evaluated using the information and concepts provided.

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## Notation

| | |
|---|---|
| $a$ | parameter of Fisher–Tippett type 1 distribution |
| $A$ | gross floor area |
| $B_X$, $B_Y$ | mean response coefficient in $X$ and $Y$ directions |
| $C$ | dynamic amplitude |
| $C_M$ | aerodynamic base moment coefficient |
| $C_X$, $C_Y$ | root mean square response coefficient in $X$ and $Y$ directions |
| $d$ | shape factor for Weibull distribution |
| $D$ | diameter |
| $E(g)$ | expected value of $g$ |
| $f$, $n$ | frequency |
| $f_0$ | natural frequency |
| $f_X$, $f_Y$ | natural frequency in $X$ and $Y$ directions |
| $F$ | mean force |
| $g$ | statistical quantity relating largest value to standard deviation |
| $H$ | height of building |
| $K$, $K_X$, $K_Y$ | exponent of r.m.s. response |
| $n$ | number of uncorrelated samples in a year |
| $p(\overline{V}, \alpha)$ | probability density of mean wind speed $\overline{V}$ at azimuth angle $\alpha$ |
| $S_F(f)$, $S_U(f)$ | power spectral density of force $(F)$ and velocity $(u)$ |
| $T$ | sample duration time |
| $U$ | mean velocity |
| $\overline{U}_g$ | mean wind speed at top of building |
| $V_1$, $V_0$ | reduced velocities |
| $\overline{V}$ | mean wind speed |
| $V(t)$ | modulating process describing mean wind speed |
| $X$ | displacement |
| $\overline{X}$, $\overline{Y}$ | mean displacement in $X$ and $Y$ directions |
| $\hat{X}$ | peak $X$ displacement |
| $Z_g$ | gradient height |
| $\alpha$ | azimuth angle |
| $\beta$ | exponent of Weibull distribution |
| $\phi(t)$ | stationary process describing gust fluctuations |
| $\rho_{\mathrm{air}}$ | density of air |
| $\rho_{\mathrm{bldg}}$ | density of a building |
| $\zeta$ | structural damping |
| $\sigma_X$, $\sigma_Y$ | root-mean-square value in $X$ and $Y$ directions |
| $\lvert\chi(f)\rvert^2$ | aerodynamic admittance |
| $\mu$ | parameter of Fisher–Tippett type 1 distribution |

### Appendix. Building properties used in wind tunnel experiments

Average density = 120 kg/m³ (7.5 lb/ft³).
Height = 658 mm (25.9 in).
$\sqrt{A}$ = 154 mm (6.06 in).
Natural frequency in $Y$ direction $f_Y = 8.49 \pm 0.01$ Hz.
Fraction of critical damping = $0.01 \pm 0.005$.
Frequency ratio $f_x/f_y = 0.978 \pm 0.003$.

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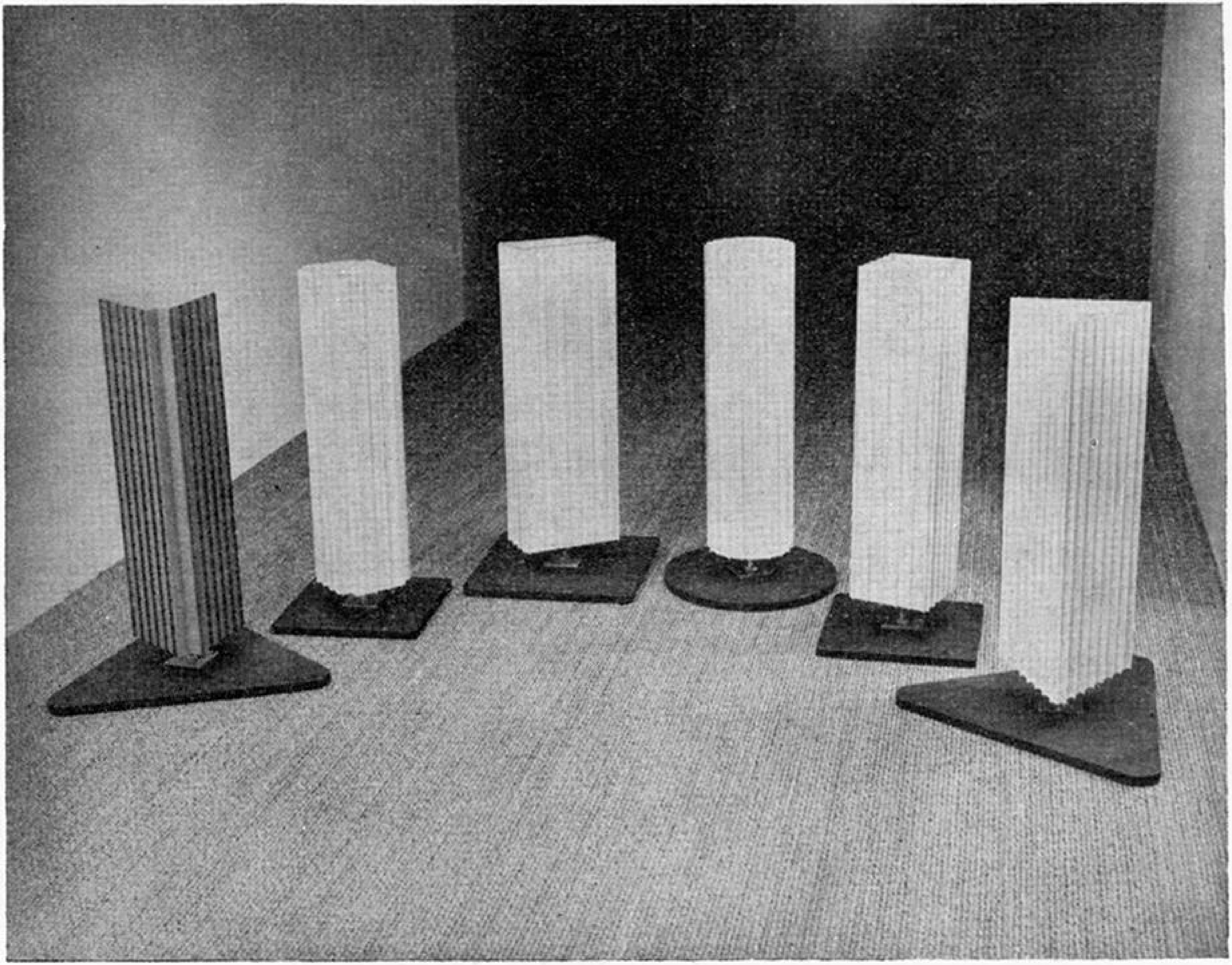

FIGURE 1. View of six building shapes studied in the boundary-layer wind-tunnel laboratory.


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